Calculus Summer 2020

DVHS (AP or RIO)

Name :

Welcome!
Congratulations on reaching this advanced level of mathematics. Calculus is unlike the mathematics you have already studied, and yet it is built on a foundation of those concepts and applications.

Things to Know: Before beginning a study of Calculus, you should have studied the topics of algebra, geometry, trigonometry, analytic geometry, and elementary functions. You should have a working understanding of functions such as linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise defined functions. This includes the properties, graphs, and algebraic manipulations of these functions. You should be familiar with the following vocabulary of functions: domain, range, odd and even functions, periodic, symmetry, zeros, intercepts, and more. Also, you MUST know the values of the trig functions at the critical points around the unit circle, without needing to sketch the entire circle!

Equipment: You are expected to have the basic supplies of mathematics at hand… pencil, paper, graph paper, straightedge, a graphing calculator, and at least occasional access to the internet.

Instructions for this Summer Work Packet: You are certainly welcome to work on this packet at any time. I would recommend, however, that you visit this packet to work your final solutions **during LATE JULY**, so that the review is fresh in your mind as you return to school in August. We will begin our year with a day or two to discuss this packet, and then **our first test grade will be based on this work**. Also, the packet with your detailed work will be collected for a grade.

NOTE: This packet is NOT designed for you to do your work on these pages. Please organize and label your work in a logical, orderly fashion before you turn it in. Give each topic its own section. Separate the pages of this packet and staple to the front of each section of your work.

This is your first impression to me as a mathematician. Make it good.

Resources: The websites below may be helpful as you review each topic. You may want to bookmark them, as they will also be helpful throughout the school year.

http://www.purplemath.com/modules/index.htm

http://tutorial.math.lamar.edu

http://www.clarku.edu/~djoyce/trig/

http://home.earthlink.net/~djbach/precalc.html

http://www.mecca.org/%7Ehalfacre/MATH/pfirstreview.html

http://www.wcpss.net/success-series/

http://college.hmco.com/mathematics/larson/calculus_early/4e/resources.html

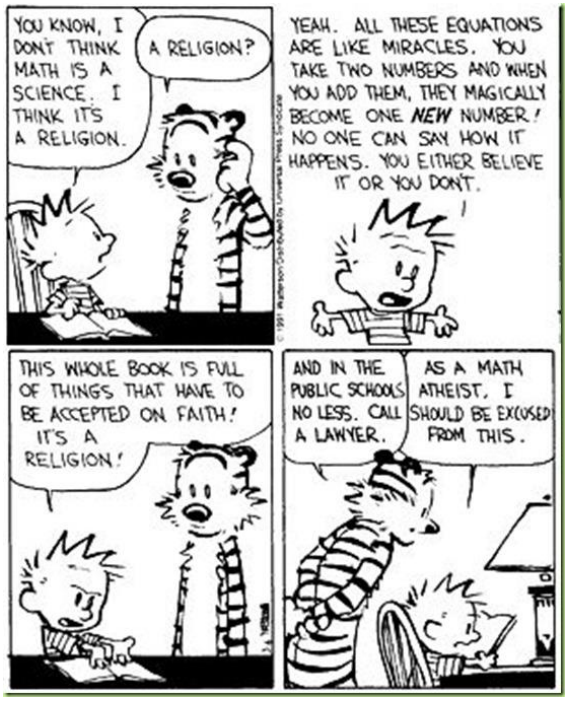
http://teachers.henrico.k12.va.us/math/HCPSAlgebra2/

If you are still struggling…

Please feel free to contact me directly at my school email:

Virginia.johnson@dvusd.org

I will be checking in at least once a week, and more frequently towards the end of summer. With your email, please include your full name, the number of the question you are asking about, and the details of what you have already tried. Often a picture of your work communicates better than a written description.



I. Algebra

Factoring, expansion, simplify expressions, simplify roots, and exponential expressions, evaluate logarithms, solve for any variable, e^x

A. Factor each completely.

1. $100x^2 - 9y^2$	2. $5x^3y^2 + 25x^2y^3 + 10x^3y^4$	3. $27a^3 + b^3$
4. $y^2 + 3y - 28$	5. $6x^2 - 17x - 14$	6. $8x^3 - 125$

B. Expand each, using the binomial theorem, not repeated multiplication.

1. $(x - 2y)^5$	2. $(2x + 3y)^3$	3. $(x + y)^4$

C. Simplify each expression.

1. $\frac{3x^2+10x+8}{6x^2+17x+10}$	2. $\frac{1}{x+h} - \frac{1}{x}$	3. $\frac{x^3-8}{x-2}$	4. $\frac{5-x}{x^2-25}$
5. $\frac{2x^2+5x-12}{x^2-16}$	6. $\frac{\frac{1}{3+x} - \frac{1}{3}}{x}$	7. $\frac{\frac{2}{x^2}}{\frac{10}{x^5}}$	8. $\frac{2x}{x^2-6x+9} - \frac{1}{x+1} - \frac{8}{x^2-2x-3}$

D. Simplify roots and exponential expressions

1. $\frac{\sqrt{x}}{x}$	2. $\left(5a^{\frac{2}{3}}\right)\left(4a^{\frac{2}{3}}\right)$	3. $\frac{4xy^{-2}}{12x^{\frac{-1}{3}}y^{-5}}$
4. $27^{\frac{2}{3}}$	5. $\left(4x^{\frac{5}{3}}\right)^{\frac{3}{2}}$	6. $x^{\frac{3}{2}}\left(x + x^{\frac{5}{2}} - x^2\right)$

E. Logarithms

1. Simplify lne^7	2. Simplify $\log_{\frac{1}{2}} 8$	3. Simplify $\log_3\left(\frac{1}{3}\right)$
4. Simplify $\ln(1) + \ln(e)$	5. Solve: $\log x + \log(x - 3) = 1$	6. Solve: $27^{(2x)} = 9^{(x-3)}$

F. Solve for named variable

1. Solve for z. $4x + 10yz = 0$	2. Solve for z. $y^2 + 3yz - 8z - 4x = 0$	3. Solve for z. $\frac{z+2}{3} - \frac{2z-7}{5} = \frac{9}{5}$

G. e^x

1. Simplify: $e^{3\ln x}$	2. Simplify: $e^{\ln(3x)}$	3. Simplify: $e^{(\ln 3)x}$

II. Graph Behavior

Graph basic functions, domain, range, intercepts, end behavior, asymptotes, piecewise functions

A. Graph each. Label each axis, and any intercepts or asymptotes

1. $y = \frac{1}{x-3}$	2. $y = \sqrt[3]{x+2}$	3. $y = e^x$
4. $y = \lvert x+3 \rvert - 2$	5. $x^2 + y^2 = 25$	6. $y = x^2 - x - 6$

B. Describe the domain and range of each

1. $y = \sqrt{x-4}$	2. $y = \sqrt{x^2-4}$	3. $y = \sqrt{x} - 4$
4. $y = \sqrt{x^2+4}$	5. $y = \sqrt{4-x^2}$	6. $y = \frac{(x+3)(x-2)}{(x^2-9)\sqrt{x-5}}$

C. Find the intercepts and describe end behavior

1. $y = 2x - 5$	2. $y = x^2 + x - 2$
3. $y = x\sqrt{16-x^2}$	4. $y^2 = x^3 - 4x$

D. Find any horizontal, vertical, or slant asymptotes

1. $f(x) = \frac{x+2}{x^2(1-x)}$	2. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$
3. $f(x) = \frac{(x-2)(x+3)}{(x+1)(x^2-4)}$	4. $f(x) = \frac{x^2+5}{x^2+2}$

E. Evaluate the requested function values

Let $(x) = 2x + 1$, $g(x) = 2x^2 - 1$, and $h(x) = x^3$. Evaluate each:

1. $f(2) - g(0)$	2. $g(a+1)$	3. $f(g(2))$
4. $g(f(2))$	5. $f(g(h(x^2)))$	6. $f^{-1}(-5)$
7. $\frac{f(x+h)-f(x)}{h}$	8. $f(2g(1)+3)$	9. $h(f(b))$

F. Graph the piecewise functions

1. $f(x) = \begin{cases} x^2 & x < 0 \\ x+2 & 0 \le x \le 3 \\ 4 & x > 3 \end{cases}$	2. $f(x) = \begin{cases} 3 & \lvert x \rvert > 5 \\ x^2 & \lvert x \rvert \le 5 \end{cases}$

III. Elementary Functions

Basic algebra of functions, composition of functions, inverse functions, partial fraction decomposition, intersections, solving systems, solving equations

A. Perform each operation, and simplify as needed

1. $\left(\frac{36-x^2}{x^2-3x-18}\right)\left(\frac{x^2+9x+18}{x^2+12x+36}\right)$	2. $\frac{5}{x-2}-\frac{2}{x}$	3. $\frac{2}{x-1}-\frac{1}{1-x}$
4. $\frac{6}{x^2-9}+\frac{1}{2x-6}$	5. $\left(\frac{36-x^2}{x^2-3x-18}\right)\div\left(\frac{x^2+9x+18}{x^2+12x+36}\right)$	6. $\left(\frac{x-3}{9-x^2}\right)^2$

B. Find the composition of functions. Give the domain of the new function created.

Let $f(x)=x^2+3$, $g(x)=\sqrt{2x-5}$, and $h(x)=4x-1$

1. Find $f(h(x))$	2. Find $g(f(x))$	3. Find $h(f(g(x)))$

C. Find the inverse function, and give the domain.

1. $f(x)=\sqrt{2x-3}$	2. $g(x)=\frac{1}{1+x}$, $x\geq 0$

D. Find the partial fraction decomposition for each expression

1. $\frac{4x+34}{x^2-5x-24}$	2. $\frac{3x^2+7x-2}{x^3-x^2-2x}$

E. Find the intersection points of the two functions, in exact form.

1. $y=x^2+3x-4$ $y=5x+11$	2. $y=\cos(x)$ $y=\sin(x)$

F. Solve each system of equations

1. $2x-3y=13$ $5x+3y=1$	2. $y=x^2+4x-4$ $y=x$	3. $y=x$ $y=x^5$

G. Solve the equations given.

1. $x^2+3x-4=14$	2. $\frac{x^4-1}{x^3}=0$	3. $12x^2=3x$
4. $(x-5)^2-9=0$	5. $2x^2+5x+8=0$	6. $(x+1)^2(x-2)+(x+1)(x-2)^2=0$

IV. Inequalities

Solve and graph, use interval notation, piecewise functions

A. Solve and graph each solution

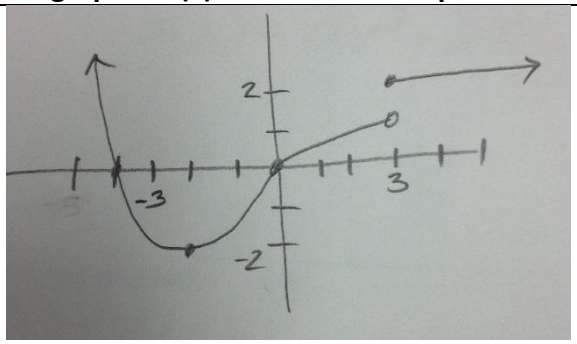
1. $x^2 - 2x - 15 \leq 0$
2. $(x-2)(x+3)^2(x-1)^3 \geq 0$
3. $|x+2| > 5$
4. $\frac{x-3}{x-1} \leq \frac{4}{x+8}$

B. Rewrite each set in interval notation

1. $\{x|-3 < x \leq 2\}$
2. $\{x|x < -4 \; OR \; x \geq 5\}$
3. $\{x|x > 2\}$
4. $\{x|x \leq 4\}$

C. Use the graph of f(x) to sketch the requested functions

f(x)



1. $-f(x)$
2. $f(|x|)$
3. $|f(x)|$
4. $|f(|x|)|$

V. Geometry

Equations of lines, slope, midpoint, distance, area, volume, similar triangles

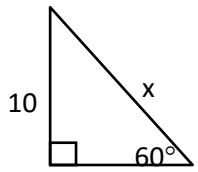
A. Find the equation for each line

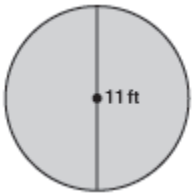
1. slope = - 2, through (3,4)	2. through (1, - 3) and (- 5, 2)	3. undefined slope, through (4, 2)
4. parallel to y = 3x + 5, through (6.3)	5. perpendicular to y = ½ x – 4, through (10, - 2)	6. Perpendicular bisector of the segment from (1, 5) to (9, 17)

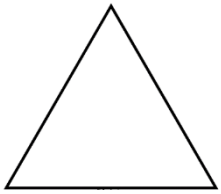
B. Find the slope, midpoint, and distance for each line segment

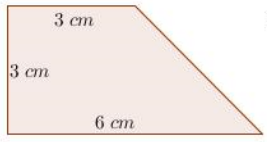
1. (- 2, 3) to (4, - 5)	2. (0,0) to (5 , - 12)	3. (5, A), to (8, 3A)

C. Compute the area of each figure

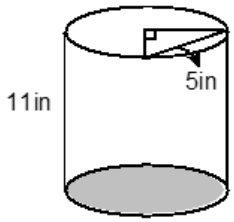
1. 

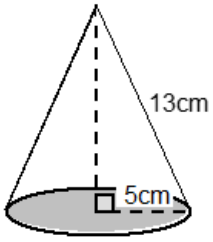
2. 

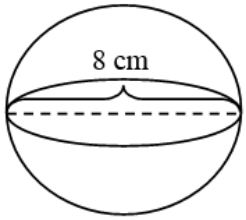
3. 
5 cm on each side.

4. 

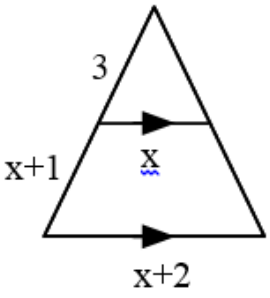
D. Compute the volume of each figure

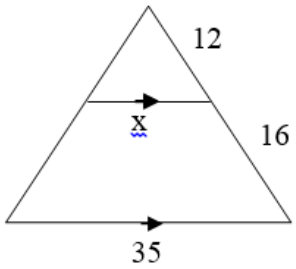
1. 

2. 

3. 

E. Compute x, and the missing measurements in each figure

1. 

2. 

VI. Trigonometry/Analytic Geometry

Unit circle values, graphs, solve trig equations, basic identities, degrees and radians, right triangle solutions

A. Give the exact value for each. You will need to be able to do these from memory, and quickly.

1. $\sin(0)$	2. $\cos\left(\frac{\pi}{6}\right)$	3. $tan\left(\frac{\pi}{4}\right)$	4. $sec\left(\frac{3\pi}{4}\right)$	5. $csc\left(\frac{5\pi}{6}\right)$	6. $\sin\left(\frac{5\pi}{6}\right)$
7. $tan\left(\frac{\pi}{6}\right)$	8. $\cos\left(\frac{5\pi}{6}\right)$	9. $tan\left(\frac{3\pi}{4}\right)$	10. $\sin\left(\frac{\pi}{6}\right)$	11. $\cos\left(\frac{3\pi}{4}\right)$	12. $\cos(0)$
13. $\cos\left(\frac{7\pi}{6}\right)$	14. $sec\left(\frac{5\pi}{6}\right)$	15. $\cos\left(\frac{-\pi}{6}\right)$	16. $tan\left(\frac{\pi}{2}\right)$	17. $csc\left(\frac{5\pi}{4}\right)$	18. $\sin\left(\frac{\pi}{4}\right)$

B. Graph each trig function. Label the axes and any asymptotes.

1. $y = \sin(x)$	2. $y = \cos(x)$
3. $y = \tan(x)$	4. $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$

C. Solve each equation, in exact form

1. $\sin(2x) = \cos(x)$	2. $(\sin(x))^2 = \frac{1}{2}$
3. $2\cos(x) = \sqrt{3}$	4. $2(\cos(x))^2 - 1 - \cos(x) = 0$

D. Complete the basic identities

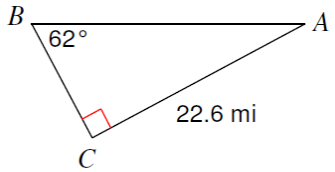
1. $sin^2(x) +$ _____ = 1	4. $\cos(2x) =$
2. $1 + tan^2(x) =$ _____	$\cos(2x) =$
3. $\sin(2x) =$	$\cos(2x) =$
	There are three different forms…

E. Convert the following between degrees and radians

1. $\frac{5\pi}{6}$ radians =	2. $\frac{3\pi}{4}$ radians =	3. $\frac{7\pi}{6}$ radians =
4. 45^o =	5. - 17^o =	6. 237^o =

F. Solve the right triangle problems.

1.



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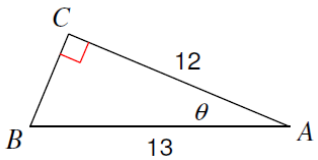


3. Devon is standing 100 feet from the Clock Tower and sees a bird land on the top of the tower. If the **angle of elevation** from Devon to the top of the Clock Tower is close to 84.6°, how tall is the tower?

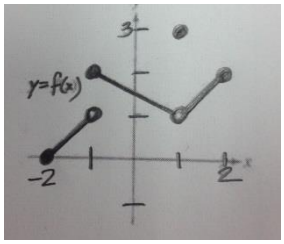
4. Two girls are standing 100 feet apart. They both see a seagull in the air between them. The angles of elevation from the girls to the bird are 20° and 45°, respectively. How high up is the seagull?

VII. Calculus

Limits (graph, table, function, at ∞, = ∞), difference quotient, average rate of change, instantaneous rate of change, equation of tangent line, power rule for derivatives

A. Evaluate each limit, using the graph of f(x) at the right

1. $\lim_{x\to -1^-} f(x)$	2. $\lim_{x\to -1^{\mp}} f(x)$	3. $\lim_{x\to -1} f(x)$	4. f(-1)
5. $\lim_{x\to 0^-} f(x)$	6. $\lim_{x\to 0^{\mp}} f(x)$	7. $\lim_{x\to 0} f(x)$	8. f(0)
9. $\lim_{x\to 1^-} f(x)$	10. $\lim_{x\to 1^{\mp}} f(x)$	11. $\lim_{x\to 1} f(x)$	12. f(1)



B. Evaluate the limit of the difference quotient for each function: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

1. $f(x) = x^2 - 2x$	2. $f(x) = x^3 + 5$

C. Find the average rate of change for each interval

1. $f(x) = 3x^2 + 4x - 2$ on the interval $[-1,5]$	2. $g(x) = -x^2 - 2x + 6$ on the interval $[-3,-1]$

D. Find the instantaneous rate of change at each value

1. $f(x) = 3x^2 - 5x + 2$, where x = 2	2. $g(x) = (2x + 5)(3x - 2)$, where x = 1

E. Find an equation for the tangent line to the function and the indicated value of x.

1. $f(x) = x^2$, where x = 1	2. $g(x) = 4x^3 + 2x^2 - 5$, where x = - 1

F. Find the derivative of each function, and simplify as needed.

1. $f(x) = 2x^3 - 4x + 5$	2. $h(x) = (3x + 5)^4$
3. $g(x) = \frac{5}{s^3}$	4. $f(x) = 5\sqrt[3]{x^4} + 4x^{\frac{5}{2}}$

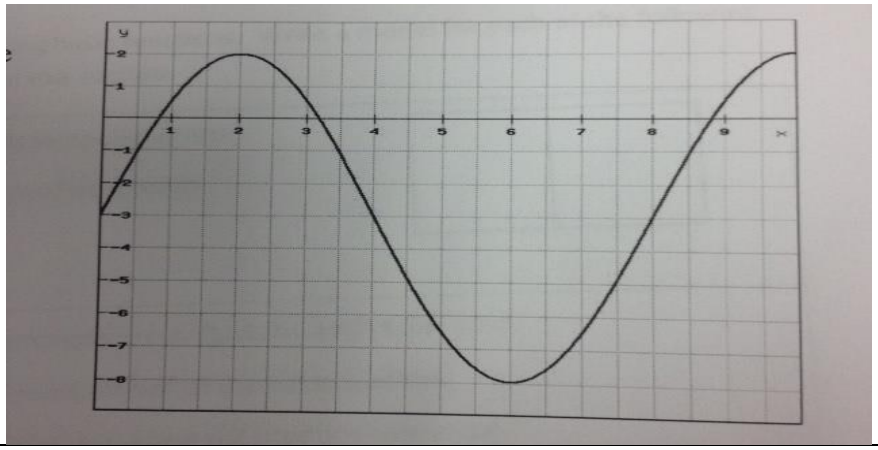
VIII. Modeling/Problem Solving

Modeling with functions, geometric graph, problem solving, new topics

A. A local rental company rents tractors for $33 for up to 3 hours and an additional $10 per hour for each hour after that up to a total of 8 hours. State a piecewise function that expresses the company's rental fees for a tractor based on the number of hours it is rented.

B. Solve the following equation for x: $\log_4(9x^2) = \log_2(5x - 8)$. Show your steps clearly.

C. Find an equation to represent the function graphed. Show all the work and logic leading to your answer.



D. Suppose that a javelin is thrown with a trajectory modeled by $h(x) = -0.1x^2 + 1.2x + 1.5$, where h(x) is the height of the javelin when it has traveled a horizontal distance of x meters. How far away from its starting point will the javelin land? How high will it have gone? Show the process you used to reach your answer.

E. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (which share one edge). What dimensions should be used so that the total enclosed area is a maximum? What will that area be?

F. A camera is mounted at a point 300 feet from the base of a rocket launching pad. The rocket rises vertically when launched. Express the distance, x, traveled by the rocket as a function of the camera elevation angle, Θ. Find the distance traveled by the rocket when the angle of elevation is 20 degrees.

Calculus BC Students ONLY

IN ADDITION TO THE CALCULUS SUMMER PACKET, ALSO DO THESE:

Show all your steps…Show a clear understanding of the process, not just a correct answer.

1. Find $\frac{dy}{dx}$.

a) $y = (4x^3 + 1)^3$ b) $2y^3 - xy + 3x^2 = 12$ c) $y = (2x)(\tan(3x))^3$

2. Find the area bounded by: $f(x) = cosx$, the x-axis, the y-axis, and $x = \frac{\pi}{4}$

3. Find the volume created by revolving the area in problem 3 about the line y = -1.

4. Find the derivative, and factor out the common factors, if any: $f(x) = (5x+2)^{\frac{2}{7}}(x^3 - 1)$

5. Find $f'(x)$ given that $f(x) = \ln\left[\frac{(6x+5)^9(3x)}{5(x-4)^3}\right]$

6. Your teacher has just informed you that the function representing your grade in calculus is positive, has a negative first derivative, and a negative second derivative. Describe your grade and how it is changing.

In two weeks, you conference with your teacher again and find that your grade function is still positive, has a negative first derivative, and a positive second derivative. What has changed, and is it a change for the better?

7. Show the STEPS to integrate: $\int 5x^2 \sin(x^3)\, dx$