CHAPTER 7
Sampling Distributions

7.3
Sample Means

The Practice of Statistics, 5th Edition
Starnes, Tabor, Yates, Moore
Sample Means

Learning Objectives

After this section, you should be able to:

✓ FIND the mean and standard deviation of the sampling distribution of a sample mean. CHECK the 10% condition before calculating the standard deviation of a sample mean.

✓ EXPLAIN how the shape of the sampling distribution of a sample mean is affected by the shape of the population distribution and the sample size.

✓ If appropriate, use a Normal distribution to CALCULATE probabilities involving sample means.
The Sampling Distribution of $\bar{x}$

When we record quantitative variables we are interested in other statistics such as the median or mean or standard deviation. Sample means are among the most common statistics.

Consider the mean household earnings for samples of size 100. Compare the population distribution on the left with the sampling distribution on the right. What do you notice about the shape, center, and spread of each?
The Sampling Distribution of $\bar{x}$

When we choose many SRSs from a population, the sampling distribution of the sample mean is centered at the population mean $\mu$ and is less spread out than the population distribution. Here are the facts.

**Sampling Distribution of a Sample Mean**

Suppose that $\bar{x}$ is the mean of an SRS of size $n$ drawn from a large population with mean $\mu$ and standard deviation $s$. Then:

- The **mean** of the sampling distribution of $\bar{x}$ is $\bar{x} = \mu$.
- The **standard deviation** of the sampling distribution of $\bar{x}$ is

$$\bar{x} = \frac{s}{\sqrt{n}}$$

as long as the **10% condition** is satisfied: $n \leq (1/10)N$.

**Note**: These facts about the mean and standard deviation of $\bar{x}$ are true no matter what shape the population distribution has.
Sampling From a Normal Population

We have described the mean and standard deviation of the sampling distribution of the sample mean $\bar{x}$ but not its shape. That's because the shape of the distribution of $\bar{x}$ depends on the shape of the population distribution.

In one important case, there is a simple relationship between the two distributions. If the population distribution is Normal, then so is the sampling distribution of $\bar{x}$. *This is true no matter what the sample size is.*

**Sampling Distribution of a Sample Mean from a Normal Population**

Suppose that a population is Normally distributed with mean and standard deviation $\mu$ and $\sigma$. Then the sampling distribution of $\bar{x}$ has the Normal distribution with mean $\mu$ and standard deviation $\sigma / \sqrt{n}$, provided that the 10% condition is met.
The Central Limit Theorem

Most population distributions are not Normal. What is the shape of the sampling distribution of sample means when the population distribution isn’t Normal?

It is a remarkable fact that as the sample size increases, the distribution of sample means changes its shape: it looks less like that of the population and more like a Normal distribution!

When the sample is large enough, the distribution of sample means is very close to Normal, *no matter what shape the population distribution has*, as long as the population has a finite standard deviation.

Draw an SRS of size $n$ from any population with mean $\mu$ and finite standard deviation $\sigma$. The central limit theorem (CLT) says that when $n$ is large, the sampling distribution of the sample mean $\bar{x}$ is approximately Normal.
The Central Limit Theorem

Consider the strange population distribution from the Rice University sampling distribution applet.

Describe the shape of the sampling distributions as n increases. What do you notice?
The Central Limit Theorem

As the previous example illustrates, even when the population distribution is very non-Normal, the sampling distribution of the sample mean often looks approximately Normal with sample sizes as small as $n = 25$.

**Normal/Large Condition for Sample Means**

If the population distribution is Normal, then so is the sampling distribution of $\bar{x}$. This is true no matter what the sample size $n$ is.

If the population distribution is not Normal, the central limit theorem tells us that the sampling distribution of $\bar{x}$ will be approximately Normal in most cases if $n \geq 30$.

The central limit theorem allows us to use Normal probability calculations to answer questions about sample means from many observations even when the population distribution is not Normal.
The Sampling Distribution of $\bar{x}$
Sample Means

Section Summary

In this section, we learned how to…

✓ FIND the mean and standard deviation of the sampling distribution of a sample mean. CHECK the 10% condition before calculating the standard deviation of a sample mean.

✓ EXPLAIN how the shape of the sampling distribution of a sample mean is affected by the shape of the population distribution and the sample size.

✓ If appropriate, use a Normal distribution to CALCULATE probabilities involving sample means.