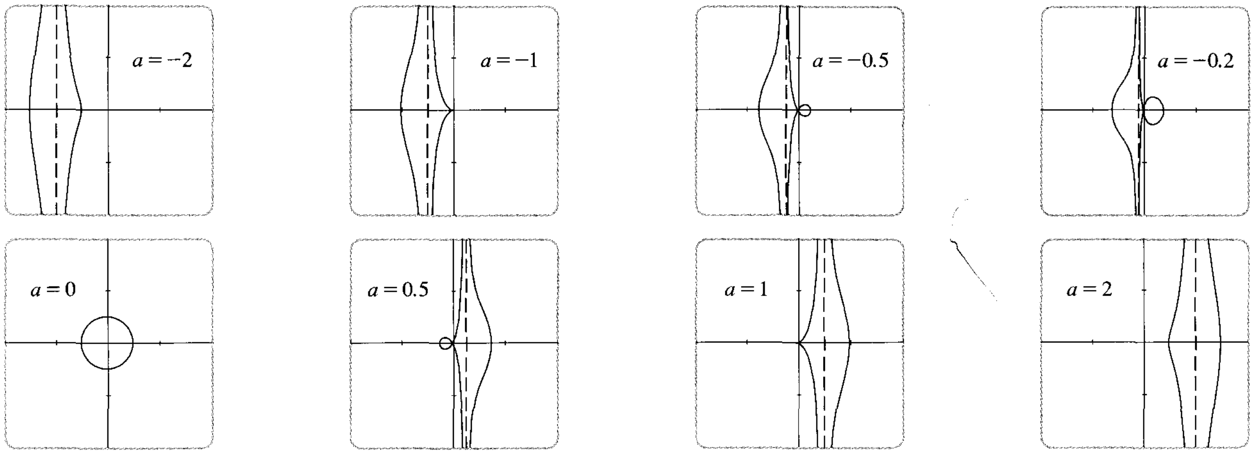


FIGURE 14 Members of the family $x = a + \cos t$, $y = a\tan t + \sin t$, all graphed in the viewing rectangle $[-4, 4]$ by $[-4, 4]$

When $a < -1$, both branches are smooth; but when a reaches -1, the right branch acquires a sharp point, called a *cusp*. For a between -1 and 0 the cusp turns into a loop, which becomes larger as a approaches 0. When $a = 0$, both branches come together and form a circle (see Example 2). For a between 0 and 1, the left branch has a loop, which shrinks to become a cusp when $a = 1$. For $a > 1$, the branches become smooth again, and as a increases further, they become less curved. Notice that the curves with a positive are reflections about the y-axis of the corresponding curves with a negative.

These curves are called **conchoids of Nicomedes** after the ancient Greek scholar Nicomedes. He called them conchoids because the shape of their outer branches resembles that of a conch shell or mussel shell. □

10.1 Exercises

1–6 □
(a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a Cartesian equation of the curve.

1. $x = 2t + 4, \quad y = t - 1$
2. $x = 3 - t, \quad y = 2t - 3, \quad -1 \leqslant t \leqslant 4$
3. $x = 1 - 2t, \quad y = t^2 + 4, \quad 0 \leqslant t \leqslant 3$
4. $x = t^2, \quad y = 6 - 3t$
5. $x = \sqrt{t}, \quad y = 1 - t$
6. $x = t^2, \quad y = t^3$

7–15 □
(a) Eliminate the parameter to find a Cartesian equation of the curve.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

7. $x = \sin\theta, \quad y = \cos\theta, \quad 0 \leqslant \theta \leqslant \pi$
8. $x = 2\cos\theta, \quad y = \frac{1}{2}\sin\theta, \quad 0 \leqslant \theta \leqslant 2\pi$
9. $x = \sin^2\theta, \quad y = \cos^2\theta$
10. $x = 2\cos\theta, \quad y = \sin^2\theta$
11. $x = e^t, \quad y = e^{-t}$
12. $x = \ln t, \quad y = \sqrt{t}, \quad t \geqslant 1$
13. $x = \tan\theta + \sec\theta, \quad y = \tan\theta - \sec\theta, \quad -\pi/2 < \theta < \pi/2$
14. $x = \cos t, \quad y = \cos 2t$
15. $x = \cosh t, \quad y = \sinh t$

16–21 □ Describe the motion of a particle with position (x, y) as t varies in the given interval.

16. $x = 4 - 4t, \quad y = 2t + 5, \quad 0 \leqslant t \leqslant 2$
17. $x = \cos \pi t, \quad y = \sin \pi t, \quad 1 \leqslant t \leqslant 2$

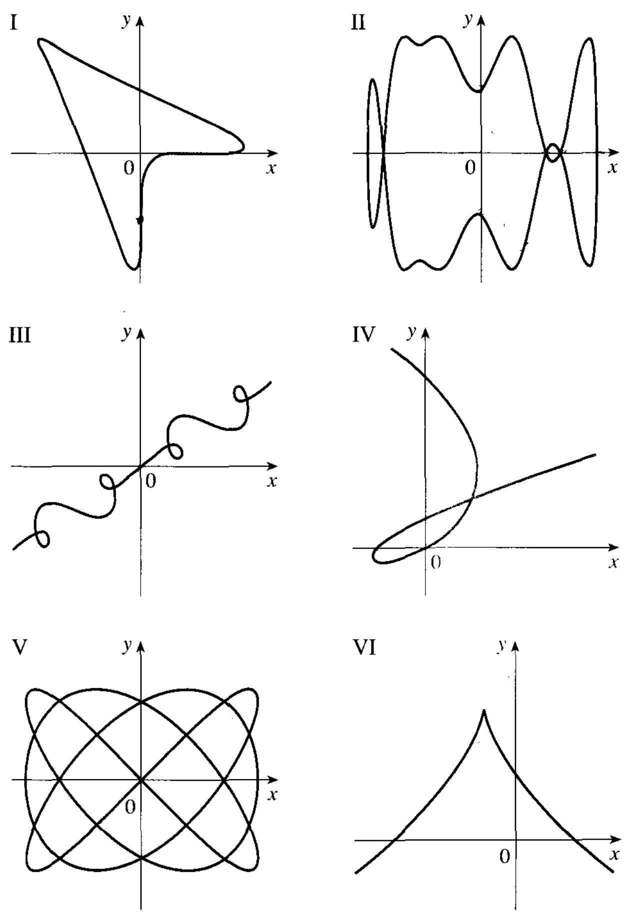
18. $x = 2 + \cos t, \quad y = 3 + \sin t, \quad 0 \leqslant t \leqslant 2\pi$

19. $x = 2 \sin t, \quad y = 3 \cos t, \quad 0 \leqslant t \leqslant 2\pi$

20. $x = \cos^2 t, \quad y = \cos t, \quad 0 \leqslant t \leqslant 4\pi$

21. $x = \tan t, \quad y = \cot t, \quad \pi/6 \leqslant t \leqslant \pi/3$

22. Match the parametric equations with the graphs labeled I–VI. Give reasons for your choices. (Do not use a graphing device.)
(a) $x = t^3 - 2t, \quad y = t^2 - t$
(b) $x = t^3 - 1, \quad y = 2 - t^2$
(c) $x = \sin 3t, \quad y = \sin 4t$
(d) $x = t + \sin 2t, \quad y = t + \sin 3t$
(e) $x = \sin(t + \sin t), \quad y = \cos(t + \cos t)$
(f) $x = \cos t, \quad y = \sin(t + \sin 5t)$



23–25 □ Graph x and y as functions of t and observe how x and y increase or decrease as t increases. Use these observations to make a rough sketch by hand of the parametric curve. Then use a graphing device to check your sketch.

23. $x = 3(t^2 - 3), \quad y = t^3 - 3t$

24. $x = \cos t, \quad y = \tan^{-1} t$

25. $x = t^4 - 1, \quad y = t^3 + 1$

26. Graph the curves $y = x^5$ and $x = y(y - 1)^2$ and find their points of intersection correct to one decimal place.

27. Graph the curve $x = y - 3y^3 + y^5$.

28. (a) Show that the parametric equations

$$x = x_1 + (x_2 - x_1)t \qquad y = y_1 + (y_2 - y_1)t$$

where $0 \leqslant t \leqslant 1$, describe the line segment that joins the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.
(b) Find parametric equations to represent the line segment from $(-2, 7)$ to $(3, -1)$.

29. Find parametric equations for the path of a particle that moves along the circle $x^2 + (y - 1)^2 = 4$ in the following manner:
(a) Once around clockwise, starting at $(2, 1)$
(b) Three times around counterclockwise, starting at $(2, 1)$
(c) Halfway around counterclockwise, starting at $(0, 3)$

30. Graph the semicircle traced by the particle in Exercise 29(c).

31. (a) Find parametric equations for the ellipse $x^2/a^2 + y^2/b^2 = 1$. [*Hint:* Modify the equations of a circle in Example 2.]
(b) Use these parametric equations to graph the ellipse when $a = 3$ and $b = 1, 2, 4$, and 8.
(c) How does the shape of the ellipse change as b varies?

32. Find three different sets of parametric equations to represent the curve $y = x^3$, $x \in \mathbb{R}$.

33. Derive Equations 1 for the case $\pi/2 < \theta < \pi$.

34. Let P be a point at a distance d from the center of a circle of radius r. The curve traced out by P as the circle rolls along a straight line is called a **trochoid**. (Think of the motion of a point on a spoke of a bicycle wheel.) The cycloid is the special case of a trochoid with $d = r$. Using the same parameter θ as for the cycloid and assuming the line is the x-axis and $\theta = 0$ when P is at one of its lowest points, show that the parametric equations of the trochoid are

$$x = r\theta - d \sin \theta \qquad y = r - d \cos \theta$$

Sketch the trochoid for the cases $d < r$ and $d > r$.

35. If a and b are fixed numbers, find parametric equations for the set of all points P determined as shown in the figure, using the angle θ as the parameter. Then eliminate the parameter and identify the curve.