(13, −110) is the lowest point on the curve. We can now be confident that there are no hidden maximum or minimum points.

(b) To find the area of the loop we need to know the parameter values that correspond to the rightmost point P on the loop, where the curve crosses itself. Zooming in toward P and using the cursor, we find that its coordinates are approximately (1.497, 22.25). The corresponding parameter values are the solutions of the equation

$$x(t) = t^2 + t + 1 = 1.497$$

The quadratic formula gives $t \approx -1.36$ and 0.36. We find the area of the loop by subtracting the area under the bottom part of the loop from the area under the top part of the loop. So the approximate area of the loop is

$$A \approx \int_{-0.5}^{0.36} (3t^4 - 8t^3 - 18t^2 + 25)(2t + 1)\,dt$$

$$- \int_{-0.5}^{-1.36} (3t^4 - 8t^3 - 18t^2 + 25)(2t + 1)\,dt$$

Combining these two integrals, we get

$$A \approx \int_{-1.36}^{0.36} (3t^4 - 8t^3 - 18t^2 + 25)(2t + 1)\,dt \approx 3.6$$

□

10.2 Exercises

1–4 □ Find dy/dx.

1. $x = t - t^3, \quad y = 2 - 5t$

2. $x = \sqrt{t} - t, \quad y = t^3 - t$

3. $x = t \ln t, \quad y = \sin^2 t$

4. $x = te^t, \quad y = t + e^t$

5–8 □ Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

5. $x = t^2 + t, \ y = t^2 - t; \quad t = 0$

6. $x = 2t^2 + 1, \ y = \frac{1}{3}t^3 - t; \quad t = 3$

7. $x = e^{\sqrt{t}}, \ y = t - \ln t^2; \quad t = 1$

8. $x = t \sin t, \ y = t \cos t; \quad t = \pi$

9–10 □ Find an equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.

9. $x = e^t, \ y = (t - 1)^2; \quad (1, 1)$

10. $x = 5 \cos t, \ y = 5 \sin t; \quad (3, 4)$

11–12 □ Find an equation of the tangent(s) to the curve at the given point. Then graph the curve and the tangent(s).

11. $x = 2 \sin 2t, \ y = 2 \sin t; \quad (\sqrt{3}, 1)$

12. $x = \sin t, \ y = \sin(t + \sin t); \quad (0, 0)$

13–18 □ Find dy/dx and d^2y/dx^2.

13. $x = t^4 - 1, \quad y = t - t^2$

14. $x = t^3 + t^2 + 1, \quad y = 1 - t^2$

15. $x = \sin \pi t, \quad y = \cos \pi t$

16. $x = 1 + \tan t, \quad y = \cos 2t$

17. $x = e^{-t}, \quad y = te^{2t}$

18. $x = 1 + t^2, \quad y = t \ln t$

19–22 □ Find the points on the curve where the tangent is horizontal or vertical. ~~Then use an analysis of the intervals in which the~~ curve rises and falls, as in Example 2, to sketch ~~the curve~~.

19. $x = t(t^2 - 3), \quad y = 3(t^2 - 3)$

20. $x = t^3 - 3t^2, \quad y = t^3 - 3t$

21. $x = \dfrac{3t}{1 + t^3}, \quad y = \dfrac{3t^2}{1 + t^3}$

22. $x = a(\cos \theta - \cos^2\theta), \quad y = a(\sin \theta - \sin \theta \cos \theta)$

23. Use a graph to estimate the coordinates of the leftmost point on the curve $x = t^4 - t^2$, $y = t + \ln t$. Then use calculus to find the exact coordinates.

24. Try to estimate the coordinates of the highest point and the leftmost point on the curve $x = te^t$, $y = te^{-t}$. Then find the exact coordinates. What are the asymptotes of this curve?

25–26 □ Graph the curve in a viewing rectangle that displays all the important aspects of the curve.

25. $x = t^4 - 2t^3 - 2t^2, \quad y = t^3 - t$

26. $x = t^4 + 4t^3 - 8t^2, \quad y = 2t^2 - t$

27. Show that the curve $x = \cos t$, $y = \sin t \cos t$ has two tangents at $(0, 0)$ and find their equations. Sketch the curve.

28. At what point does the curve $x = 1 - 2\cos^2 t$, $y = (\tan t)(1 - 2\cos^2 t)$ cross itself? Find the equations of both tangents at that point.

29. (a) Find the slope of the tangent line to the trochoid $x = r\theta - d\sin\theta$, $y = r - d\cos\theta$ in terms of θ. (See Exercise 34 in Section 10.1.)
(b) Show that if $d < r$, then the trochoid does not have a vertical tangent.

30. (a) Find the slope of the tangent to the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ in terms of θ. (Astroids are explored in the Laboratory Project on page 648.)
(b) At what points is the tangent horizontal or vertical?
(c) At what points does the tangent have slope 1 or -1?

31. At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with equations $x = -7t$, $y = 12t - 5$?

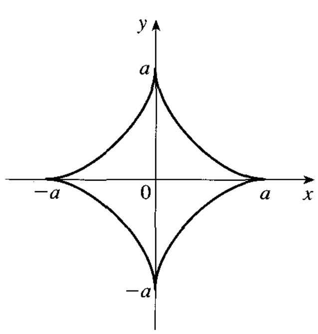
32. Find equations of the tangents to the curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ that pass through the point $(4, 3)$.

33. Find the area bounded by the curve $x = \cos t$, $y = e^t$, $0 \leq t \leq \pi/2$, and the lines $y = 1$ and $x = 0$.

34. Find the area bounded by the curve $x = t - 1/t$, $y = t + 1/t$ and the line $y = 2.5$.

35. Use the parametric equations of an ellipse, $x = a\cos\theta$, $y = b\sin\theta$, $0 \leq \theta \leq 2\pi$, to find the area that it encloses.

36. Find the area of the region enclosed by the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$. (Astroids are explored in the Laboratory Project on page 648.)



37. Find the area under one arch of the trochoid of Exercise 34 in Section 10.1 for the case $d < r$.

38. Let $\mathcal{R}$ be the region enclosed by the loop of the curve in Example 2.
(a) Find the area of $\mathcal{R}$.
(b) If $\mathcal{R}$ is rotated about the x-axis, find the volume of the resulting solid.
(c) Find the centroid of $\mathcal{R}$.

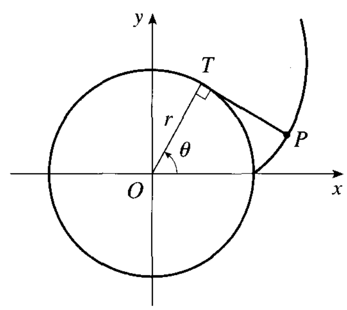
39. Estimate the area of the region enclosed by each loop of the curve

$$x = \sin t - 2\cos t \qquad y = 1 + \sin t \cos t$$

40. If f' is continuous and $f'(t) \neq 0$ for $a \leq t \leq b$, show that the parametric curve $x = f(t)$, $y = g(t)$, $a \leq t \leq b$, can be put in the form $y = F(x)$. [*Hint:* Show that f^{-1} exists.]

41. A string is wound around a circle and then unwound while being held taut. The curve traced by the point P at the end of the string is called the **involute** of the circle. If the circle has radius r and center O and the initial position of P is $(r, 0)$, and if the parameter θ is chosen as in the figure, show that parametric equations of the involute are

$$x = r(\cos\theta + \theta\sin\theta) \qquad y = r(\sin\theta - \theta\cos\theta)$$



42. A cow is tied to a silo with radius r by a rope just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow.