SOLUTION Figure 18 shows computer-drawn graphs for various values of c. For $c > 1$ there is a loop that decreases in size as c decreases. When $c = 1$ the loop disappears and the curve becomes the cardioid that we sketched in Example 7. For c between 1 and $\frac{1}{2}$ the cardioid's cusp is smoothed out and becomes a "dimple." When c decreases from $\frac{1}{2}$ to 0, the limaçon is shaped like an oval. This oval becomes more circular as $c \to 0$, and when $c = 0$ the curve is just the circle $r = 1$.

□ In Exercise 55 you are asked to prove analytically what we have discovered from the graphs in Figure 18.

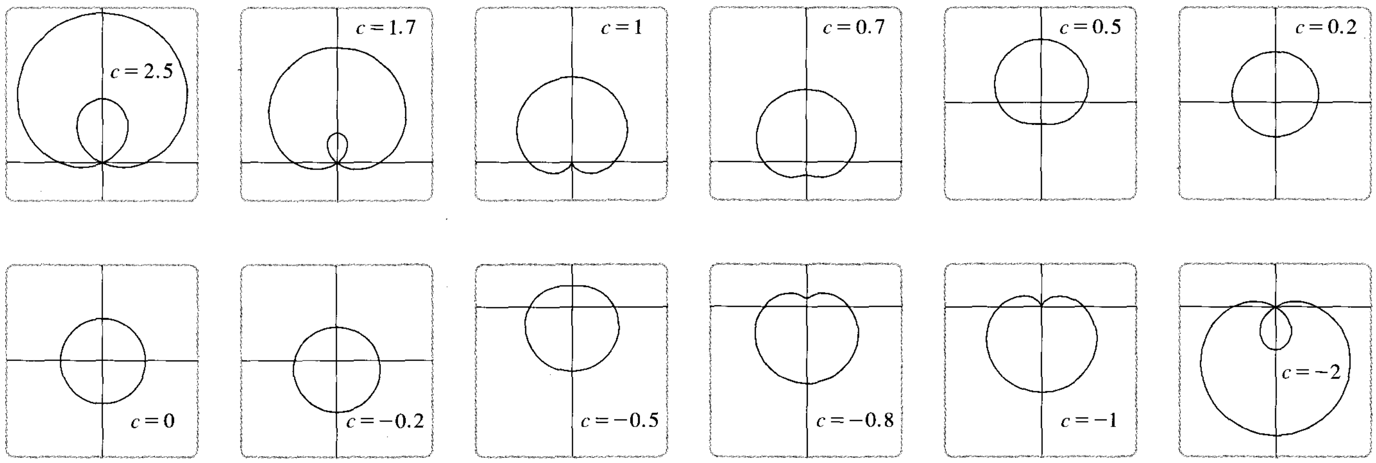


FIGURE 18
Members of the family of limaçons $r = 1 + c \sin \theta$

The remaining parts of Figure 18 show that as c becomes negative, the shapes change in reverse order. In fact, these curves are reflections about the horizontal axis of the corresponding curves with positive c. □

10.4 Exercises

1–2 □ Plot the point whose polar coordinates are given. Then find two other pairs of polar coordinates of this point, one with $r > 0$ and one with $r < 0$.

1. (a) $(1, \pi/2)$ (b) $(-2, \pi/4)$ (c) $(3, 2)$

2. (a) $(3, 0)$ (b) $(2, -\pi/7)$ (c) $(-1, -\pi/2)$

3–4 □ Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

3. (a) $(3, \pi/2)$ (b) $(2\sqrt{2}, 3\pi/4)$ (c) $(-1, \pi/3)$

4. (a) $(2, 2\pi/3)$ (b) $(4, 3\pi)$ (c) $(-2, -5\pi/6)$

5–6 □ The Cartesian coordinates of a point are given.
(i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
(ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

5. (a) $(1, 1)$ (b) $(2\sqrt{3}, -2)$

6. (a) $(-1, -\sqrt{3})$ (b) $(-2, 3)$

7–12 □ Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.

7. $r > 1$

8. $0 \leq \theta < \pi/4$

9. $0 \leq r \leq 2, \quad \pi/2 \leq \theta \leq \pi$

10. $1 \leq r < 3, \quad -\pi/4 \leq \theta \leq \pi/4$

11. $2 < r < 3, \quad 5\pi/3 \leq \theta \leq 7\pi/3$

12. $-1 \leq r \leq 1, \quad \pi/4 \leq \theta \leq 3\pi/4$

13. Find the distance between the points with polar coordinates $(1, \pi/6)$ and $(3, 3\pi/4)$.

14. Find a formula for the distance between the points with polar coordinates (r_1, θ_1) and (r_2, θ_2).

15–20 □ Find a Cartesian equation for the curve described by the given polar equation.

15. $r = 2$

16. $r \cos \theta = 1$

17. $r = 3 \sin \theta$

18. $r = 1/(1 + 2 \sin \theta)$

19. $r^2 = \sin 2\theta$

20. $r^2 = \theta$

21–26 □ Find a polar equation for the curve represented by the given Cartesian equation.

21. $y = 5$ $r \sin\theta = 5$
22. $y = 2x - 1$
23. $x^2 + y^2 = 25$
24. $x^2 = 4y$
25. $2xy = 1$
26. $x^2 - y^2 = 1$

27–28 □ For each of the described curves, decide if the curve would be more easily given by a polar equation or a Cartesian equation. Then write an equation for the curve.

27. (a) A line through the origin that makes an angle of $\pi/6$ with the positive x-axis
(b) A vertical line through the point $(3, 3)$

28. (a) A circle with radius 5 and center $(2, 3)$
(b) A circle centered at the origin with radius 4

29–32 □ Sketch the curve of the polar equation by first converting it to a Cartesian equation.

29. $r = -2 \sin\theta$
30. $r = 2\sin\theta + 2\cos\theta$
31. $r = \csc\theta$
32. $r = \tan\theta \sec\theta$

33–50 □ Sketch the curve with the given equation.

33. $r = 5$
34. $\theta = 3\pi/4$
35. $r = \sin\theta$
36. $r = -3\cos\theta$
37. $r = 2(1 - \sin\theta)$
38. $r = 1 - 3\cos\theta$
39. $r = \theta, \quad \theta \geqslant 0$
40. $r = \theta/2, \quad -4\pi \leqslant \theta \leqslant 4\pi$
41. $r = 1/\theta$
42. $r = \sqrt{\theta}$
43. $r = \sin 2\theta$
44. $r = 2\cos 3\theta$
45. $r = 2\cos 4\theta$
46. $r = \sin 5\theta$
47. $r^2 = 4\cos 2\theta$
48. $r^2 = \sin 2\theta$
49. $r = 2\cos(3\theta/2)$
50. $r^2\theta = 1$

51. Show that the polar curve $r = 4 + 2\sec\theta$ (called a **conchoid**) has the line $x = 2$ as a vertical asymptote by showing that $\lim_{r\to\pm\infty} x = 2$. Use this fact to help sketch the conchoid.

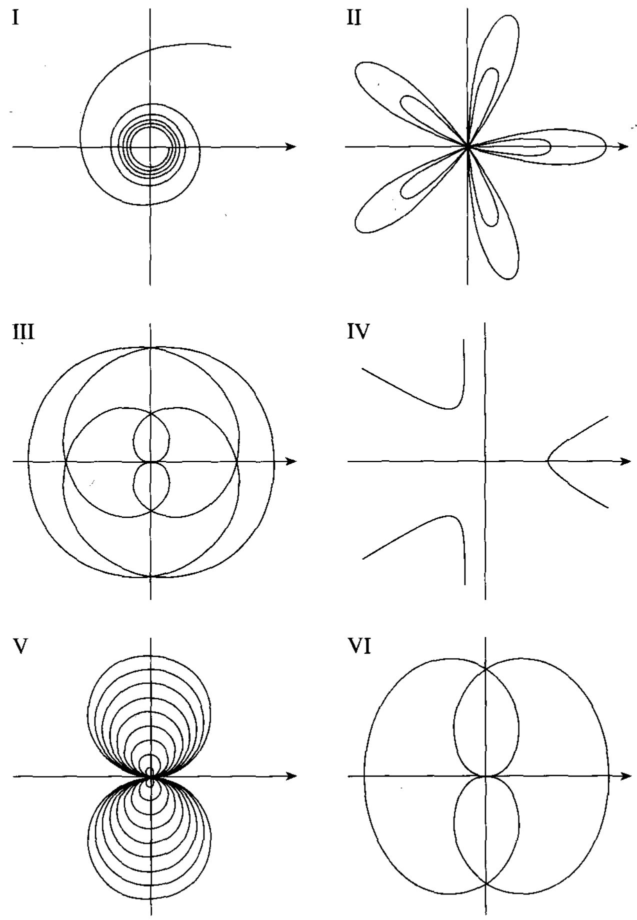
52. Show that the curve $r = 2 - \csc\theta$ (also a conchoid) has the line $y = -1$ as a horizontal asymptote by showing that $\lim_{r\to\pm\infty} y = -1$. Use this fact to help sketch the conchoid.

53. Show that the curve $r = \sin\theta \tan\theta$ (called a **cissoid of Diocles**) has the line $x = 1$ as a vertical asymptote. Show also that the curve lies entirely within the vertical strip $0 \leqslant x < 1$. Use these facts to help sketch the cissoid.

54. Sketch the curve $(x^2 + y^2)^3 = 4x^2y^2$.

55. (a) In Example 11 the graphs suggest that the limaçon $r = 1 + c\sin\theta$ has an inner loop when $|c| > 1$. Prove that this is true, and find the values of θ that correspond to the inner loop.
(b) From Figure 18 it appears that the limaçon loses its dimple when $c = \frac{1}{2}$. Prove this.

56. Match the polar equations with the graphs labeled I–VI. Give reasons for your choices. (Don't use a graphing device.)
(a) $r = \sin(\theta/2)$
(b) $r = \sin(\theta/4)$
(c) $r = \sec(3\theta)$
(d) $r = \theta\sin\theta$
(e) $r = 1 + 4\cos 5\theta$
(f) $r = 1/\sqrt{\theta}$



57–62 □ Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

57. $r = 3\cos\theta, \quad \theta = \pi/3$
58. $r = \cos\theta + \sin\theta, \quad \theta = \pi/4$
59. $r = 1/\theta, \quad \theta = \pi$
60. $r = \ln\theta, \quad \theta = e$
61. $r = 1 + \cos\theta, \quad \theta = \pi/6$
62. $r = \sin 3\theta, \quad \theta = \pi/6$

63–68 □ Find the points on the given curve where the tangent line is horizontal or vertical.

63. $r = 3\cos\theta$
64. $r = \cos\theta + \sin\theta$
65. $r = 1 + \cos\theta$
66. $r = e^\theta$
67. $r = \cos 2\theta$
68. $r^2 = \sin 2\theta$