We could evaluate this integral by multiplying and dividing the integrand by $\sqrt{2 - 2\sin\theta}$, or we could use a computer algebra system. In any event, we find that the length of the cardioid is $L = 8$. □

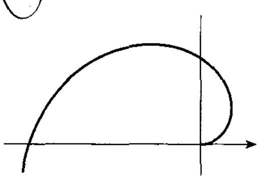
10.5 Exercises

1–4 □ Find the area of the region that is bounded by the given curve and lies in the specified sector.

1. $r = \sqrt{\theta}$, $0 \leqslant \theta \leqslant \pi/4$
2. $r = e^{\theta/2}$, $\pi \leqslant \theta \leqslant 2\pi$
3. $r = \sin\theta$, $\pi/3 \leqslant \theta \leqslant 2\pi/3$
4. $r = \sqrt{\sin\theta}$, $0 \leqslant \theta \leqslant \pi$

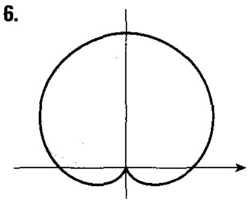
5–8 □ Find the area of the shaded region.

5.



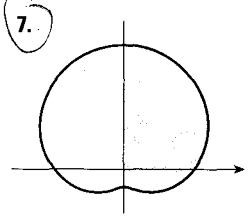
$r = \theta$

6.



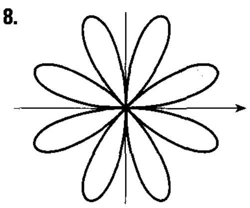
$r = 1 + \sin\theta$

7.



$r = 4 + 3\sin\theta$

8.



$r = \sin 4\theta$

9–14 □ Sketch the curve and find the area that it encloses.

9. $r = 5\sin\theta$
10. $r = 3(1 + \cos\theta)$
11. $r^2 = 4\cos 2\theta$
12. $r^2 = \sin 2\theta$
13. $r = 4 - \sin\theta$
14. $r = \sin 3\theta$

15. Graph the curve $r = 2 + \cos 6\theta$ and find the area that it encloses.

16. The curve with polar equation $r = 2\sin\theta\cos^2\theta$ is called a **bifolium**. Graph it and find the area that it encloses.

17–22 □ Find the area of the region enclosed by one loop of the curve.

17. $r = \sin 2\theta$
18. $r = 4\sin 3\theta$
19. $r = 3\cos 5\theta$
20. $r = 2\cos 4\theta$
21. $r = 1 + 2\sin\theta$ (inner loop)
22. $r = 2 + 3\cos\theta$ (inner loop)

23–28 □ Find the area of the region that lies inside the first curve and outside the second curve.

23. $r = 1 - \cos\theta$, $r = \frac{3}{2}$
24. $r = 1 - \sin\theta$, $r = 1$
25. $r = 4\sin\theta$, $r = 2$
26. $r = 3\cos\theta$, $r = 2 - \cos\theta$
27. $r = 3\cos\theta$, $r = 1 + \cos\theta$
28. $r = 1 + \cos\theta$, $r = 3\cos\theta$

29–34 □ Find the area of the region that lies inside both curves.

29. $r = \sin\theta$, $r = \cos\theta$
30. $r = \sin 2\theta$, $r = \sin\theta$
31. $r = \sin 2\theta$, $r = \cos 2\theta$
32. $r^2 = 2\sin 2\theta$, $r = 1$
33. $r = 3 + 2\sin\theta$, $r = 2$
34. $r = a\sin\theta$, $r = b\cos\theta$, $a > 0$, $b > 0$

35. Find the area inside the larger loop and outside the smaller loop of the limaçon $r = \frac{1}{2} + \cos\theta$.

36. Graph the hippopede $r = \sqrt{1 - 0.8\sin^2\theta}$ and the circle $r = \sin\theta$ and find the exact area of the region that lies inside both curves.

37–42 □ Find all points of intersection of the given curves.

37. $r = \sin\theta$, $r = \cos\theta$
38. $r = 2$, $r = 2\cos 2\theta$
39. $r = \cos\theta$, $r = 1 - \cos\theta$
40. $r = \cos 3\theta$, $r = \sin 3\theta$
41. $r = \sin\theta$, $r = \sin 2\theta$
42. $r^2 = \sin 2\theta$, $r^2 = \cos 2\theta$

43. The points of intersection of the cardioid $r = 1 + \sin\theta$ and the spiral loop $r = 2\theta$, $-\pi/2 \leqslant \theta \leqslant \pi/2$, can't be found exactly. Use a graphing device to find the approximate values of θ at which they intersect. Then use these values to estimate the area that lies inside both curves.

44. Use a graph to estimate the values of θ for which the curves $r = 3 + \sin 5\theta$ and $r = 6\sin\theta$ intersect. Then estimate the area that lies inside both curves.

45–50 □ Find the length of the polar curve.

45. $r = 5\cos\theta$, $0 \leqslant \theta \leqslant 3\pi/4$
46. $r = e^{2\theta}$, $0 \leqslant \theta \leqslant 2\pi$
47. $r = 2^{\theta}$, $0 \leqslant \theta \leqslant 2\pi$
48. $r = \theta$, $0 \leqslant \theta \leqslant 2\pi$
49. $r = \theta^2$, $0 \leqslant \theta \leqslant 2\pi$
50. $r = 1 + \cos\theta$

51–52 □ Use a calculator or computer to find the length of the loop correct to four decimal places.

51. One loop of the four-leaved rose $r = \cos 2\theta$

52. The loop of the conchoid $r = 4 + 2 \sec \theta$

53–54 □ Graph the curve and find its length.

53. $r = \cos^4(\theta/4)$ **54.** $r = \cos^2(\theta/2)$

55. (a) Use Formula 10.3.5 to show that the area of the surface generated by rotating the polar curve

$$r = f(\theta) \qquad a \leqslant \theta \leqslant b$$

(where f' is continuous and $0 \leqslant a < b \leqslant \pi$) about the polar axis is

$$S = \int_a^b 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

(b) Use the formula in part (a) to find the surface area generated by rotating the lemniscate $r^2 = \cos 2\theta$ about the polar axis.

56. (a) Find a formula for the area of the surface generated by rotating the polar curve $r = f(\theta)$, $a \leqslant \theta \leqslant b$ (where f' is continuous and $0 \leqslant a < b \leqslant \pi$), about the line $\theta = \pi/2$.
(b) Find the surface area generated by rotating the lemniscate $r^2 = \cos 2\theta$ about the line $\theta = \pi/2$.

10.6 Conic Sections

In this section we give geometric definitions of parabolas, ellipses, and hyperbolas and derive their standard equations. They are called **conic sections**, or **conics**, because they result from intersecting a cone with a plane as shown in Figure 1.

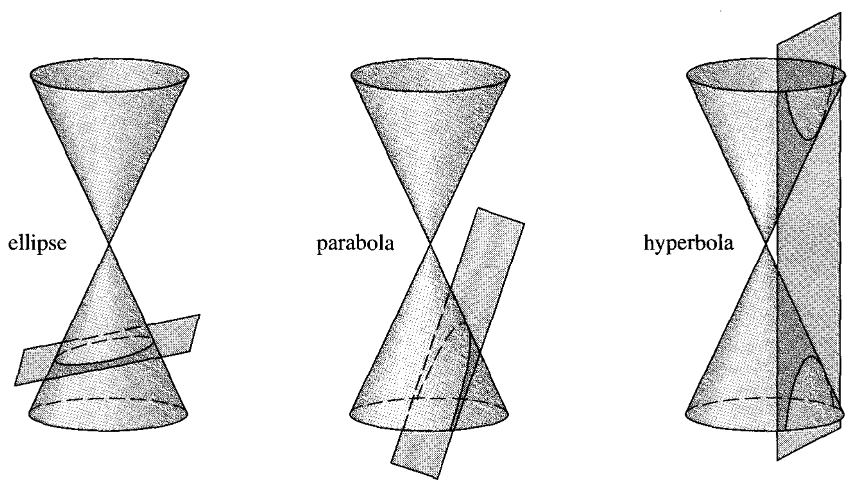


FIGURE 1
Conics

Parabolas

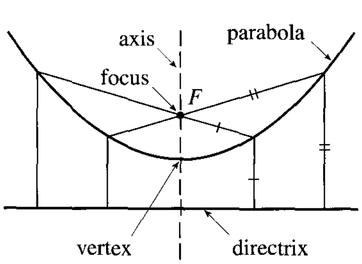


FIGURE 2

A **parabola** is the set of points in a plane that are equidistant from a fixed point F (called the **focus**) and a fixed line (called the **directrix**). This definition is illustrated by Figure 2. Notice that the point halfway between the focus and the directrix lies on the parabola; it is called the **vertex**. The line through the focus perpendicular to the directrix is called the **axis** of the parabola.

In the 16th century Galileo showed that the path of a projectile that is shot into the air at an angle to the ground is a parabola. Since then, parabolic shapes have been used in designing automobile headlights, reflecting telescopes, and suspension bridges. (See Problem 16 on page 274 for the reflection property of parabolas that makes them so useful.)