We must integrate between the appropriate y-values, $y = -2$ and $y = 4$. Thus

$$A = \int_{-2}^{4} (x_R - x_L)\,dy$$

$$= \int_{-2}^{4} \left[(y + 1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy$$

$$= \int_{-2}^{4} \left(-\tfrac{1}{2}y^2 + y + 4\right) dy$$

$$= -\frac{1}{2}\left(\frac{y^3}{3}\right) + \frac{y^2}{2} + 4y\,\Big]_{-2}^{4}$$

$$= -\tfrac{1}{6}(64) + 8 + 16 - \left(\tfrac{4}{3} + 2 - 8\right) = 18$$

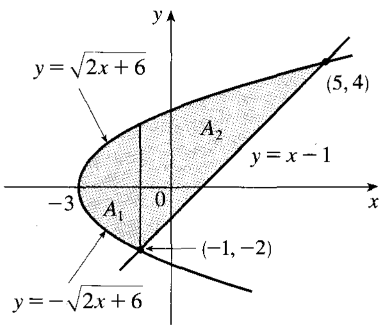


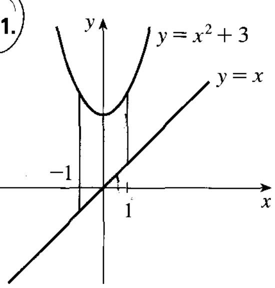
FIGURE 14

We could have found the area in Example 6 by integrating with respect to x instead of y, but the calculation is much more involved. It would have meant splitting the region in two and computing the areas labeled A_1 and A_2 in Figure 14. The method we used in Example 6 is *much* easier.

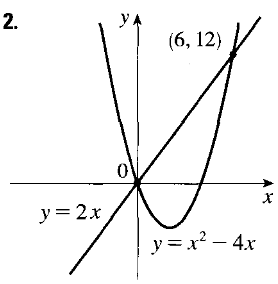
6.1 Exercises

1–4 □ Find the area of the shaded region.

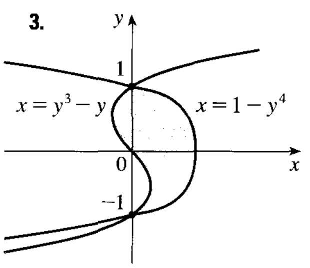
1.



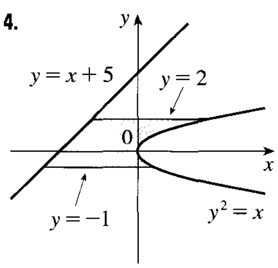
2.



3.



4.



5–26 □ Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5. $y = x + 1,\quad y = 9 - x^2,\quad x = -1,\quad x = 2$

6. $y = \sin x,\quad y = e^x,\quad x = 0,\quad x = \pi/2$

7. $y = x,\quad y = x^2$

8. $y = x^2,\quad y = x^4$

9. $y = 1/x,\quad y = 1/x^2,\quad x = 2$

10. $y = 1 + \sqrt{x},\quad y = (3 + x)/3$

11. $y = x^2,\quad y^2 = x$

12. $y = x,\quad y = \sqrt[3]{x}$

13. $y = 4x^2,\quad y = x^2 + 3$

14. $y = x^3 - x,\quad y = 3x$

15. $y = x + 1,\quad y = (x - 1)^2,\quad x = -1,\quad x = 2$

16. $y = x^2 + 1,\quad y = 3 - x^2,\quad x = -2,\quad x = 2$

17. $y^2 = x,\quad x - 2y = 3$

18. $y = 1/x,\quad x = 0,\quad y = 1,\quad y = 2$

19. $x = 1 - y^2,\quad x = y^2 - 1$

20. $y = \cos x,\quad y = \sec^2 x,\quad x = -\pi/4,\quad x = \pi/4$

21. $y = \cos x,\quad y = \sin 2x,\quad x = 0,\quad x = \pi/2$

22. $y = \sin x,\quad y = \sin 2x,\quad x = 0,\quad x = \pi/2$

23. $y = \cos x,\quad y = 1 - 2x/\pi$

24. $y = |x|,\quad y = x^2 - 2$

25. $y = x^2,\quad y = 2/(x^2 + 1)$

26. $y = \sin \pi x,\quad y = x^2 - x,\quad x = 2$

27–28 □ Use calculus to find the area of the triangle with the given vertices.

27. $(0, 0),\quad (2, 1),\quad (-1, 6)$

28. $(0, 5),\quad (2, -2),\quad (5, 1)$

29–30 □ Evaluate the integral and interpret it as the area of a region. Sketch the region.

29. $\int_{-1}^{1} |x^3 - x|\,dx$

30. $\int_{0}^{\pi} \left| \sin x - \frac{2}{\pi}x \right| dx$

31–32 □ Use the Midpoint Rule with $n = 4$ to approximate the area of the region bounded by the given curves.

31. $y = \sqrt{1 + x^3}$, $y = 1 - x$, $x = 2$

32. $y = x \tan x$, $y = x$

33–36 □ Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

33. $y = x^2$, $y = 2\cos x$

34. $y = x^4$, $y = 3x - x^3$

35. $y = x^2$, $y = e^{-x^2}$

36. $y = e^x$, $y = 2 - x^2$

37–38 □ Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then use the Midpoint Rule with $n = 4$ to approximate the area of the region bounded by the curves.

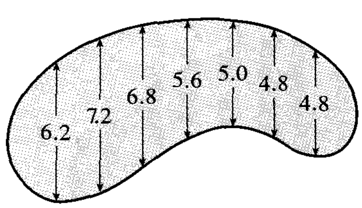
37. $y = 1 + 3x - 2x^2$, $y = \sqrt{1 + x^4}$

38. $y = x^2 - x$, $y = \sin(x^2)$

39. Racing cars driven by Chris and Kelly are side by side at the start of a race. The table shows the velocities of each car (in miles per hour) during the first ten seconds of the race. Use the Midpoint Rule to estimate how much farther Kelly travels than Chris does during the first ten seconds.

t	v_C	v_K	t	v_C	v_K
0	0	0	6	69	80
1	20	22	7	75	86
2	32	37	8	81	93
3	46	52	9	86	98
4	54	61	10	90	102
5	62	71			

40. The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the figure. Use the Midpoint Rule to estimate the area of the pool.



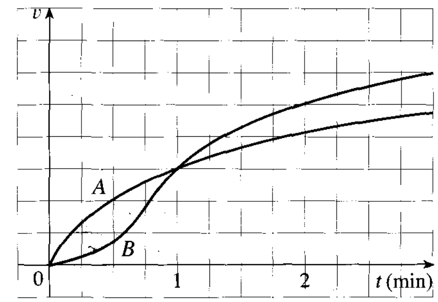
41. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.

(a) Which car is ahead after one minute? Explain.

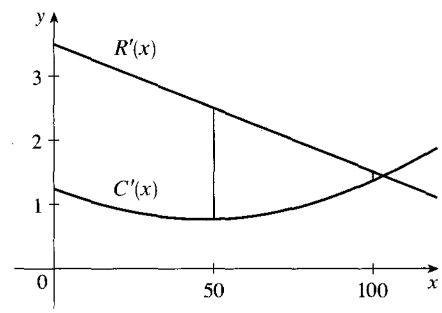
(b) What is the meaning of the area of the shaded region?

(c) Which car is ahead after two minutes? Explain.

(d) Estimate the time at which the cars are again side by side.



42. The figure shows graphs of the marginal revenue function R' and the marginal cost function C' for a manufacturer. [Recall from Section 4.8 that $R(x)$ and $C(x)$ represent the revenue and cost when x units are manufactured. Assume that R and C are measured in thousands of dollars.] What is the meaning of the area of the shaded region? Use the Midpoint Rule to estimate the value of this quantity.



43. The curve with equation $y^2 = x^2(x + 3)$ is called **Tschirnhausen's cubic**. If you graph this curve you will see that part of the curve forms a loop. Find the area enclosed by the loop.

44. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at (1, 1), and the x-axis.

45. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

46. (a) Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \leq x \leq 4$.

(b) Find the number b such that the line $y = b$ bisects the area in part (a).

47. Find the values of c such that the area of the region enclosed by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

48. Suppose that $0 < c < \pi/2$. For what value of c is the area of the region enclosed by the curves $y = \cos x$, $y = \cos(x - c)$, and $x = 0$ equal to the area of the region enclosed by the curves $y = \cos(x - c)$, $x = \pi$, and $y = 0$?

49. For what values of m do the line $y = mx$ and the curve $y = x/(x^2 + 1)$ enclose a region? Find the area of the region.