we would have obtained the integral

$$V = \int_0^h \frac{L^2}{h^2}(h - y)^2\,dy = \frac{L^2h}{3}$$

EXAMPLE 9 □ A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.

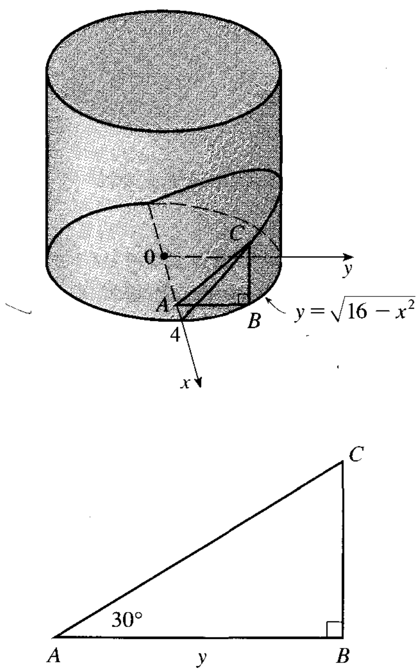


FIGURE 17

SOLUTION If we place the x-axis along the diameter where the planes meet, then the base of the solid is a semicircle with equation $y = \sqrt{16 - x^2}$, $-4 \leqslant x \leqslant 4$. A cross-section perpendicular to the x-axis at a distance x from the origin is a triangle ABC, as shown in Figure 17, whose base is $y = \sqrt{16 - x^2}$ and whose height is $|BC| = y \tan 30° = \sqrt{16 - x^2}/\sqrt{3}$. Thus, the cross-sectional area is

$$A(x) = \tfrac{1}{2}\sqrt{16 - x^2} \cdot \frac{1}{\sqrt{3}}\sqrt{16 - x^2}$$
$$= \frac{16 - x^2}{2\sqrt{3}}$$

and the volume is

$$V = \int_{-4}^{4} A(x)\,dx = \int_{-4}^{4} \frac{16 - x^2}{2\sqrt{3}}\,dx$$
$$= \frac{1}{\sqrt{3}}\int_0^4 (16 - x^2)\,dx = \frac{1}{\sqrt{3}}\left[16x - \frac{x^3}{3}\right]_0^4$$
$$= \frac{128}{3\sqrt{3}}$$

For another method see Exercise 60. □

6.2 Exercises

1–18 □ Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or "washer."

Resources / Module 7 / Volumes / Problems and Tests

1. $y = x^2$, $x = 1$, $y = 0$; about the x-axis

2. $y = e^x$, $y = 0$, $x = 0$, $x = 1$; about the x-axis

3. $y = 1/x$, $x = 1$, $x = 2$, $y = 0$; about the x-axis

4. $y = \sqrt{x - 1}$, $x = 2$, $x = 5$, $y = 0$; about the x-axis

5. $y = x^2$, $0 \leqslant x \leqslant 2$, $y = 4$, $x = 0$; about the y-axis

6. $x = y - y^2$, $x = 0$; about the y-axis

7. $y = x^2$, $y^2 = x$; about the x-axis

8. $y = \sec x$, $y = 1$, $x = -1$, $x = 1$; about the x-axis

9. $y^2 = x$, $x = 2y$; about the y-axis

10. $y = x^{2/3}$, $x = 1$, $y = 0$; about the y-axis

11. $y = x$, $y = \sqrt{x}$; about $y = 1$

12. $y = x^2$, $y = 4$; about $y = 4$

13. $y = x^4$, $y = 1$; about $y = 2$

14. $y = 1/x$, $y = 0$, $x = 1$, $x = 3$; about $y = -1$

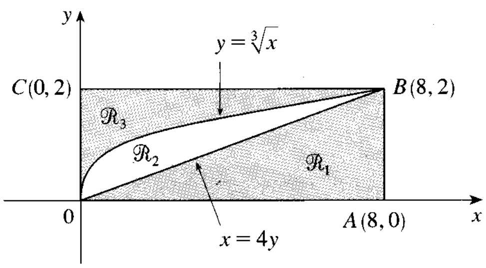
15. $x = y^2$, $x = 1$; about $x = 1$

16. $y = x$, $y = \sqrt{x}$; about $x = 2$

17. $y = x^2$, $x = y^2$; about $x = -1$

18. $y = x$, $y = 0$, $x = 2$, $x = 4$; about $x = 1$

19–30 □ Refer to the figure and find the volume generated by rotating the given region about the given line.



19. $\mathcal{R}_1$ about OA

20. $\mathcal{R}_1$ about OC

21. $\mathcal{R}_1$ about AB

22. $\mathcal{R}_1$ about BC

23. $\mathcal{R}_2$ about OA

24. $\mathcal{R}_2$ about OC

25. $\mathcal{R}_2$ about BC

26. $\mathcal{R}_2$ about AB

27. $\mathcal{R}_3$ about OA

28. $\mathcal{R}_3$ about OC

29. $\mathcal{R}_3$ about BC

30. $\mathcal{R}_3$ about AB

31–36 □ Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

31. $y = \ln x$, $y = 1$, $x = 1$; about the x-axis

32. $y = \sqrt{x - 1}$, $y = 0$, $x = 5$; about the y-axis

33. $y = 0$, $y = \sin x$, $0 \leqslant x \leqslant \pi$; about $y = 1$

34. $y = 0$, $y = \sin x$, $0 \leqslant x \leqslant \pi$; about $y = -2$

35. $x^2 - y^2 = 1$, $x = 3$; about $x = -2$

36. $2x + 3y = 6$, $(y - 1)^2 = 4 - x$; about $x = -5$

37–38 □ Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the volume of the solid obtained by rotating about the x-axis the region bounded by these curves.

37. $y = x^2$, $y = \ln(x + 1)$

38. $y = 3\sin(x^2)$, $y = e^{x/2} + e^{-2x}$

39–42 □ Each integral represents the volume of a solid. Describe the solid.

39. $\pi \int_0^{\pi/2} \cos^2 x \, dx$

40. $\pi \int_2^5 y \, dy$

41. $\pi \int_0^1 (y^4 - y^8) \, dy$

42. $\pi \int_0^{\pi/2} [(1 + \cos x)^2 - 1^2] \, dx$

43. A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 1.5 cm apart. The liver is 15 cm long and the cross-sectional areas, in square centimeters, are 0, 18, 58, 79, 94, 106, 117, 128, 63, 39, and 0. Use the Midpoint Rule to estimate the volume of the liver.

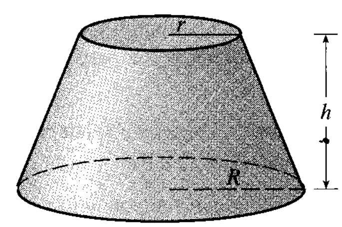
44. A log 10 m long is cut at 1-meter intervals and its cross-sectional areas A (at a distance x from the end of the log) are listed in the table. Use the Midpoint Rule with $n = 5$ to estimate the volume of the log.

x (m)	A (m^2)	x (m)	A (m^2)
0	0.68	6	0.53
1	0.65	7	0.55
2	0.64	8	0.52
3	0.61	9	0.50
4	0.58	10	0.48
5	0.59		

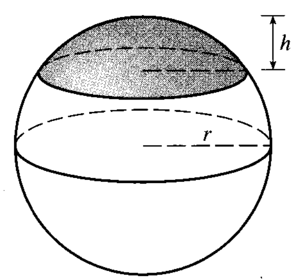
45–57 □ Find the volume of the described solid S.

45. A right circular cone with height h and base radius r

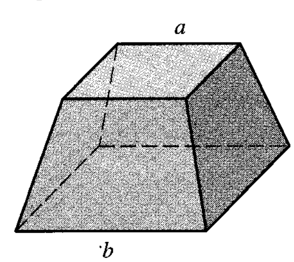
46. A frustum of a right circular cone with height h, lower base radius R, and top radius r



47. A cap of a sphere with radius r and height h



48. A frustum of a pyramid with square base of side b, square top of side a, and height h



49. A pyramid with height h and rectangular base with dimensions b and $2b$