On the other hand, the average value of the velocity function on the interval is

$$v_{\text{ave}} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)\,dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} s'(t)\,dt$$

$$= \frac{1}{t_2 - t_1}[s(t_2) - s(t_1)] \qquad \text{(by the Total Change Theorem)}$$

$$= \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \text{average velocity}$$

□

6.5 Exercises

1–8 □ Find the average value of the function on the given interval.

1. $f(x) = x^2, \quad [-1, 1]$
2. $f(x) = 1/x, \quad [1, 4]$
3. $g(x) = \cos x, \quad [0, \pi/2]$
4. $g(x) = \sqrt{x}, \quad [1, 4]$
5. $f(t) = te^{-t^2}, \quad [0, 5]$
6. $f(\theta) = \sec\theta\tan\theta, \quad [0, \pi/4]$
7. $h(x) = \cos^4 x \sin x, \quad [0, \pi]$
8. $h(r) = 3/(1 + r)^2, \quad [1, 6]$

9–12 □

(a) Find the average value of f on the given interval.
(b) Find c such that $f_{\text{ave}} = f(c)$.
(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

9. $f(x) = 4 - x^2, \quad [0, 2]$
10. $f(x) = e^x, \quad [0, 2]$
11. $f(x) = x^3 - x + 1, \quad [0, 2]$
12. $f(x) = x\sin(x^2); \quad [0, \sqrt{\pi}]$

13. If f is continuous and $\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.

14. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

15. In a certain city the temperature (in °F) t hours after 9 A.M. was approximated by the function

$$T(t) = 50 + 14\sin\frac{\pi t}{12}$$

Find the average temperature during the period from 9 A.M. to 9 P.M.

16. The temperature of a metal rod, 5 m long, is $4x$ (in °C) at a distance x meters from one end of the rod. What is the average temperature of the rod?

17. The linear density in a rod 8 m long is $12/\sqrt{x + 1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

18. If a freely falling body starts from rest, then its displacement is given by $s = \frac{1}{2}gt^2$. Let the velocity after a time T be v_T. Show that if we compute the average of the velocities with respect to t we get $v_{\text{ave}} = \frac{1}{2}v_T$, but if we compute the average of the velocities with respect to s we get $v_{\text{ave}} = \frac{2}{3}v_T$.

19. Use the result of Exercise 75 in Section 5.5 to compute the average volume of inhaled air in the lungs in one respiratory cycle.

20. The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood (see Example 7 in Section 3.3). Find the average velocity (with respect to r) over the interval $0 \leq r \leq R$. Compare the average velocity with the maximum velocity.

21. Prove the Mean Value Theorem for Integrals by applying the Mean Value Theorem for derivatives (see Section 4.2) to the function $F(x) = \int_a^x f(t)\,dt$.

22. If $f_{\text{ave}}[a, b]$ denotes the average value of f on the interval $[a, b]$ and $a < c < b$, show that

$$f_{\text{ave}}[a, b] = \frac{c - a}{b - a} f_{\text{ave}}[a, c] + \frac{b - c}{b - a} f_{\text{ave}}[c, b]$$