AP Calculus AB: Differential Equations Name Key

Calculate the general or particular solution equation for each of the following differential equations:

1) $\frac{dy}{dx} = 2xy$

Let $e^c = k$

$\int \frac{1}{y}dy = \int 2x\,dx$

$\ln|y| = x^2 + c$

$y = e^{x^2} \cdot e^c$

$y = ke^{x^2}$

2) $\frac{dy}{dx} = \frac{2+y}{x}$

Let $e^c = k$

$\int \frac{1}{2+y}dy = \int \frac{1}{x}dx$

$\ln|2+y| = \ln|x| + c$

$2+y = e^{\ln|x| + c}$

$2+y = e^{\ln|x|} \cdot e^c$

$2+y = k|x|$

$y = k|x| - 2$

3) $\frac{dy}{dx} = \frac{2y}{x} \rightarrow \frac{1}{2y}dy = \frac{1}{x}dx$

$\frac{1}{2}\int \frac{1}{y}dy = \int \frac{1}{x}dx$

$\frac{1}{2}\ln|y| = \ln|x| + c$

$\ln|y| = 2\ln|x| + c$

$y = e^{\ln|x^2|} \cdot e^c$

$y = x^2 \cdot k$

$y = kx^2$

Let $e^c = k$

4) $\frac{dy}{dx} = \frac{-2x}{y}$ with $f(1) = -1$

$\int y\,dy = \int -2x\,dx$

$\frac{1}{2}y^2 = -x^2 + c$

$y^2 = -2x^2 + c$

$y = \pm\sqrt{-2x^2 + c}$

$(-1) = (\pm\sqrt{-2+c})^2$

$1 = -2 + c$

$c = 3$

$y = -\sqrt{-2x^2 + 3}$

5) $\frac{dy}{dx} = (y-1)^2 \sin(\pi x)$

$\int \frac{dy}{(y-1)^2} = \int \sin(\pi x)\,dx$

$-(y-1)^{-1} = -\frac{1}{\pi}\cos\pi x + c$

$\frac{1}{(y-1)} = \frac{\cos\pi x + c\pi}{\pi}$

$-(y-1) = \frac{\pi}{-\cos\pi x + c\pi}$ 6)

$y - 1 = \frac{\pi}{\cos\pi x + c\pi}$

$y = \frac{\pi}{\cos\pi x + c\pi} + 1$

$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ with $f(0) = \frac{1}{2}$

$\int e^{2y}dy = \int 3x^2 dx$

$\frac{1}{2}e^{2y} = x^3 + c$

$e^{2y} = 2x^3 + c$

$2y = \ln(2x^3 + c)$

$y = \frac{1}{2}\ln(2x^3 + c)$

$\frac{1}{2} = \frac{1}{2}\ln(c)$

$e^1 = e^{\ln c}$

$c = e$

$y = \frac{1}{2}\ln(2x^3 + e)$

7) $\frac{dy}{dx} = \frac{-xy^2}{2}$ with $f(-1) = 2$

$\int \frac{1}{y^2}dy = \int -\frac{1}{2}x\,dx$

$-y^{-1} = -\frac{1}{4}x^2 + c$

$\frac{-1}{y} = -\frac{x^2}{4} + c$

$\frac{-1}{y} = \frac{-x^2 + 4c}{4}$

$-4 = y(-x^2 + 4c)$

$y = \frac{+4}{+x^2 + 4c}$

$2 = \frac{4}{1 + 4c}$

$1 + 4c = 2$

$4c = 1$

$c = 1/4$

$y = \frac{4}{x^2 + 1}$

8) $\frac{dy}{dx} = x^2(y-1)$ with $f(0) = 3$

$\int \frac{1}{y-1}dy = \int x^2 dx$

$\ln|y-1| = \frac{1}{3}x^3 + c$

$e^{\ln|y-1|} = e^{\frac{1}{3}x^3 + c}$

$y - 1 = k \cdot e^{\frac{1}{3}x^3}$

$y = ke^{\frac{1}{3}x^3} + 1$

Let $e^c = k$

$3 = k \cdot 1 + 1$

$2 = k$

$y = 2e^{\frac{1}{3}x^3} + 1$