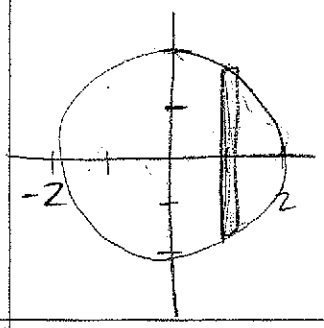
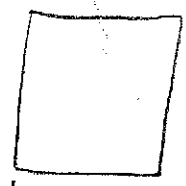
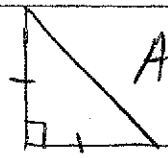
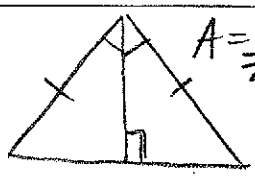
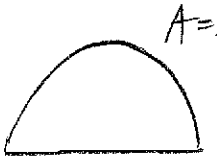
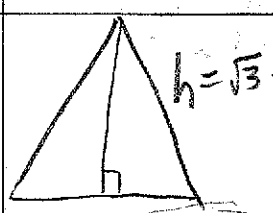
Volume by Cross Sections **Name** Key **Date**

(1) Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$ The indicated cross sections are taken perpendicular to the x-axis.

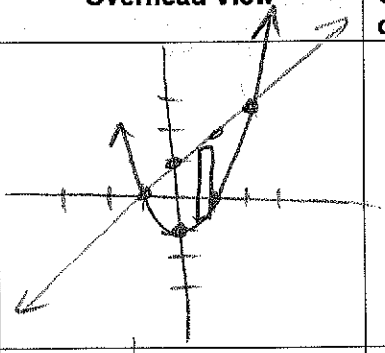
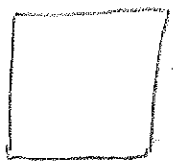
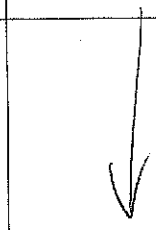
$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$

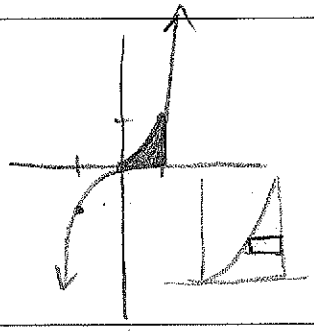
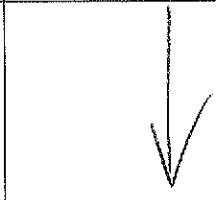
Solid	Overhead View	Cross Section with dimensions / volume	Volume Integral
a) Squares with one side in the plane of the circle		 $b = 2\sqrt{4-x^2}$ $A = b^2$	$V = \int_{-2}^{2} 4(4-x^2)\,dx =$ $42.\overline{6}$
b) Isosceles right triangles with one leg in the plane of the circle.		 $A = \frac{1}{2}b \cdot b$ $b = 2\sqrt{4-x^2}$	$V = \int_{-2}^{2} 2(4-x^2)\,dx =$ $21.\overline{3}$
c) Isosceles right triangles with the hypotenuse in the plane of the circle		 $A = \frac{1}{2}bh$ $b = 2\sqrt{4-x^2}$ $h = \sqrt{4-x^2}$	$V = \int_{-2}^{2} (4-x^2)\,dx =$ $10.\overline{6}$
d) Semicircles with the diameter in the plane of the circle		 $A = \frac{1}{2}\pi r^2$ $d = 2\sqrt{4-x^2}$ $r = \sqrt{4-x^2}$	$V = \int_{-2}^{2} \frac{1}{2}\pi(4-x^2)\,dx =$ $5.\overline{3}\pi$ OR 16.755
e) Equilateral triangles with a side in the plane of the circle		 $h = \sqrt{3}\sqrt{4-x^2}$ $b = 2\sqrt{4-x^2}$ $A = \frac{1}{2}bh$	$V = \int_{-2}^{2} \sqrt{3}(4-x^2)\,dx =$ 18.475

$A = \frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}\left(2\sqrt{4-x^2}\right)^2$

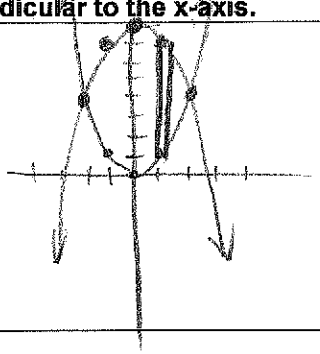
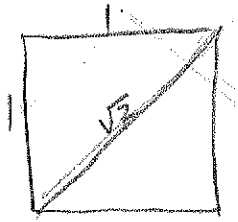
Find the volume of a solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$
With the indicated cross sections taken perpendicular to the x-axis.

Solid	Overhead View	Cross Section with dimensions / volume	Volume Integral
f) Squares with one side in the plane ~~of the circle~~		 $A = b^2$ $b = (x+1) - (x^2-1)$ $b = x+1-x^2+1$	$\int_{-1}^{2} (-x^2+x+2)^2 dx$ 8.1
g) Rectangles of height 1 unit with the base in the plane ~~of the circle~~		 $b = -x^2+x+2$ $A = 1(-x^2+x+2)$	$\int_{-1}^{2} (-x^2+x+2) dx$ 4.5

Find volume of a solid with a base bounded by $y = x^3$, $y = 0$, and $x = 1$.
The cross sections are taken perpendicular to the y-axis.

h) Squares with one side in the base		 $A = b^2$ $b = 1 - \sqrt[3]{y}$	$\int_{0}^{1} (1-\sqrt[3]{y})^2 dy$ 0.099 or 0.100
i) Semicircles with the diameters in the plane ~~of the circle~~		 $A = \frac{1}{2}\pi r^2$ $d = 1 - \sqrt[3]{y}$ $r = \frac{1-\sqrt[3]{y}}{2}$ $r^2 = \frac{1}{4}(1-\sqrt[3]{y})^2$	$\int_{0}^{1} \frac{\pi}{8}(1-\sqrt[3]{y})^2 dy$ 0.012 π or 0.039

The base of the solid is bounded by the parabolas $y=x^2$ and $y = 8 - x^2$.
The cross sections are perpendicular to the x-axis.

j) Squares with diagonals in the plane of the parabolas.		 $\frac{\sqrt{2}}{(8-x^2-x^2)} = \frac{1}{s}$ $\left(\frac{1}{2}\right)^2 \left(\frac{(8-2x^2)}{1}\right)^2$	$\int_{-2}^{2} \frac{1}{2}(8-2x^2)^2 dx$ $\frac{1}{2}(8-2x^2)(8-2x^2)$ $2(4-x^2)\cdot 2(4-x^2)$ $\int_{-2}^{2} 2(4-x^2)^2 dx$ 68.267