Linear Functions

In this 14-lesson module, students are introduced to bivariate data. Students work with functions and use their understanding of functions to model the possible relationships of bivariate data. This module is important in setting a foundation for students’ work in algebra in Grade 9 with respect to functions and statistics.

What Came Before this Module:

Students learn the concept of a function and why functions are necessary for describing geometric concepts and occurrences in everyday life. Students also learn about the important role functions play in making predictions. Students will inspect the rate of change of linear functions and conclude that the rate of change is the slope of the graph of a line. They will learn to interpret the equation \( y = mx + b \) as defining a linear function whose graph is a line. Students will also gain some experience with non-linear functions, specifically by compiling and graphing a set of ordered pairs, and then by identifying the graph as something other than a straight line.

Key Words

Association
A relationship between two variables. The tendency for two variables to vary together in a predictable way.

Two-way table
A table used to summarize data on two categorical variables. The rows of the table correspond to the possible categories for one of the variables, and the columns correspond to the possible categories for the other variable. Entries in the cells of the table indicate the number of times that a particular category combination occurs in the data set or the frequency for that combination.

Row relative frequency
In a two-way table, a row relative frequency is a cell frequency divided by the row total for that cell.

Column relative frequency
In a two-way table, a column relative frequency is a cell frequency divided by the column total for that cell.

Patterns in Scatter Plots

There is a pattern in this graph that looks as though it could be well-described by a line. Therefore, this relationship represents a negative linear relationship.

There is a pattern in this graph that looks as though it could be well-described by a line. Therefore, this relationship represents a positive linear relationship.

What Comes After this Module:

Students will begin this module with work related to the Pythagorean Theorem. Students also learn the notation related to roots and learn that in order to get the decimal expansion of a number, they must develop a deeper understanding of the long division algorithm learned in Grade 6 and Grade 7. In addition, students learn that radical expressions arise naturally in geometry and apply the Pythagorean Theorem to three-dimensional figures.

Key Common Core Standards:

Use functions to model relationships between quantities.

- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x,y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values.
- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.

- Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
- Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

How can you help at home?

- Every day, ask your child what they learned in school and ask them to show you an example.
- Ask your child what an outlier means when analyzing a set of data.
- Discuss what a graph of a linear function with a positive slope would look like and have your child draw a sketch of this relationship.
- Ask your child to create a situation that could explain part B of the graph below.

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Lesson 6: Scatter Plots

*This is a sample problem from the curriculum.

Data were collected on

x= shoe size and y= score on a reading test for 30 elementary school students. The scatter plot of these data is shown below. Does there appear to be a statistical relationship between shoe size and score on the reading test?

Possible answer: The pattern in the scatter plot appears to follow a line. As shoe sizes increase, the reading score also seems to increase. There does appear to be a statistical relationship because there is a pattern in the scatter plot.

![Scatter plot]

Explain why it is not reasonable to conclude that having big feet causes a high reading score. Can you think of a different explanation for why you might see a pattern like this?

Possible answer: You cannot conclude that just because there is a statistical relationship between shoe size and reading score that one causes the other. These data were for students completing a reading test for younger elementary school children. Older children, who would have bigger feet than younger children, would probably be able to score higher on a reading test for younger students.

Lesson 1: Modeling Linear Relationships

*This is a sample problem from the curriculum.

Problem:

A rental car company offers a rental package for a mid-size car. The cost is comprised of a fixed $30 administrative fee for the cleaning and maintenance of the car plus a rental cost of $35 per day.

1. Using x for the number of days and y for the total cost in dollars, construct a function to model the relationship between the number of days and the total cost of renting a mid-size car.

2. The same company is advertising a deal on compact car rentals. The linear function $y = 30x + 15$ can be used to model the relationship between the number of days (x) and the total cost (y) of renting a compact car.

Solution:

1. $y = 35x + 30$

2. a. $15$ is the fixed administrative fee

b. $30$ is the rental cost per day.